STRESS AND STRAIN IN A PLATE SUBJECTED TO INTENSIVE DRYING

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The study was carried out to investigate the development of a stress–strain state in a thin porous plate subjected to intensive drying. A narrow evaporation zone (an evaporation front) divides the plate into two regions with different structures and rheological behavior. The complete saturation area is described by Kelvin–Voigt's viscoelastic model, and the dry area, by Hooke's elasticity law. Shrinkage is suggested to be a function of the evaporation zone velocity. The influence of variable shrinkage on the stress distribution across the plate is studied.

The development of a stress–strain state in porous materials being dried is associated with some physicochemical processes, and its description is rather difficult.

The authors of [1-3] suggest interrelated heat transfer and stress–strain state equations with the use of irreversible thermodynamics. This process is investigated in [4] for a periodical monodisperse colloid system with a small drying rate within the framework of physicochemical mechanics.

In this study the development of a stress–strain state is investigated in a thin plate subjected to high-rate drying. It is assumed that there is a narrow evaporation zone which divides the plate into two regions with different structures and rheological states. This takes place when there is no capillary inflow because of intensive evaporation. In the central fully saturated area Kelvin–Voigt's viscoelastic model is adopted. In the peripheral area the structure formation occurred, and rigid contacts between the particles allows Hooke's elasticity law to be used.

In the evaporation zone, apart from capillary forces, viscous drag will be exerted on the particles. The particle spacing will depend on the time they are present in the evaporation zone and, consequently, on the front velocity.

For a saturated porous medium Kelvin–Voigt's model with the shear and volume viscosity will have the form

$$s_{ij} = 2G(1 + \tau\partial_t)\, e_{ij}, \quad \sigma_{ii} = 3K(1 + \tau_0\partial_t)(\varepsilon_{ii} - \Theta), \tag{1}$$

where s_{ij}, e_{ij} are the components of the stress and strain tensor deviators, respectively.

It follows from Eq. (1) that

$$\sigma_{ij} = 2G(1 + \tau\partial_t)\,\varepsilon_{ij} + \left[3K(1 + \tau_0\partial_t)(\varepsilon_{ii} - \Theta) - \frac{2}{3}G(1 + \tau\partial_t)\,\varepsilon_{ii}\delta_{ij}\right], \tag{2}$$

where $\sigma_{ij} = s_{ij} + 1/3\sigma_{ii}\delta_{ij}$, $\varepsilon_{ij} = e_{ij} + 1/3\varepsilon_{ii}\delta_{ij}$.

For a plate it is useful to exclude ε_{ii} from Eq. (2); then we will have

$$6GK(1 + \tau\partial_t)(1 + \tau_0\partial_t)(\varepsilon_{ij} - \Theta/3\delta_{ij}) = 3K(1 + \tau_0\partial_t)\,\sigma_{ij} - [3K(1 + \tau_0\partial_t) - 2G(1 + \tau\partial_t)]\,1/3\sigma_{ii}\delta_{ij}. \tag{3}$$

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The medium plane of the plate will be made coincident with the plane (x, y) of the Cartesian coordinate system. A one-dimensional drying regime will be assumed, in which the moisture content depends on the z coordinate alone. For a thin plane plate we may set

$$\sigma_{zz}=0,\quad \sigma_{xx}=\sigma_{yy}=\sigma(z),\quad \varepsilon_{xx}=\varepsilon_{yy}=\varepsilon(z),\quad \varepsilon_{zz}\neq 0. \tag{4}$$

Then, Eq. (3) reduces to the following relation

$$\dot{\sigma}+B\sigma=A(1+\tau_0\partial_t)(1+\tau\partial_t)(\varepsilon-\Theta/3), \tag{5}$$

where

$$A=\frac{18GK}{4G\tau+3K\tau_0},\quad B=\frac{4G+3K}{4G\tau+3K\tau_0}.$$

Since Eq. (5) is written for the fully saturated region, the shrinkage in it is zero ($\Theta = 0$). As was noted above, in the fully dried area the shrinkage is a function of the front velocity. This factor is included by the following relation

$$\Theta(V)=\frac{\Theta_0 V_0}{V_0+V}, \tag{6}$$

where Θ_0 is shrinkage upon slow drying.

As the evaporation front moves inward, the resistance to vapor removal increases and the drying rate goes down. It may be approximately written that the vapor flow is defined by the dried zone width and the effective vapor transfer coefficient in the porous medium [6]

$$j=\frac{[D]\,MP_s}{RT\,(h-z_*)},$$

Hence the front velocity is equal to

$$V=-\frac{dz_*}{dt}=\frac{j}{\rho}. \tag{7}$$

In order to obtain an exact solution for the vapor flow, it is necessary to solve a set of differential mass transfer equations [6].

From the compatibility conditions with Eq. (4) the equation $\partial^2\varepsilon/\partial z^2 = 0$ remains, hence

$$\varepsilon(z,\ t)=a(t)\,z+b(t).$$

For a symmetrical problem the strains are independent of $z(\varepsilon = \varepsilon(t))$.

At the start of drying ($t = 0$), the strains of the plate are equal to zero, since strain discontinuities are impossible in Kelvin–Voigt's model. In the dry region the stresses experience a finite discontinuity at $t = 0$, but in the saturated region the strains are continuous, since the dry region thickness is zero at the initial moment. Then, from Eq. (2) for σ_{zz} and σ at $t = 0$ a system of linear homogeneous equations it follows a nonzero determinant, hence follows that at the initial moment the strain rates are also zero: $\dot{\varepsilon} = \dot{\varepsilon}_{zz} = 0$.

Integration of Eq. (5) with the zero initial conditions

$$t=0:\quad \sigma=0,\quad \varepsilon=\dot{\varepsilon}=0, \tag{8}$$

gives

$$\sigma=a_0\varepsilon+a_1\dot{\varepsilon}+a_2\int_0^t\varepsilon\exp[-B(t-\tau)]\,d\tau,$$

where $a_0 = A(\tau_0 + \tau - B\tau_0\tau)$, $a_1 = A\tau_0\tau$, $a_2 = A[1 - B(\tau_0 + \tau) + B^2\tau_0\tau]$.

Thus, we have

$$\sigma(z,\ t)=\begin{cases} a_0\varepsilon+a_1\dot{\varepsilon}+a_2\displaystyle\int_0^t\varepsilon\exp[-B(t-\tau)]\,d\tau, & z<z_*,\\ \dfrac{E}{1-\nu}[\varepsilon-\Theta(z)/3], & z_*<z<h. \end{cases} \tag{9}$$

Boundary conditions at the free plate contour are of the form

$$\int_{-h}^{h} \sigma(z,\ t)\,dz = 0, \quad \int_{-h}^{h} z\sigma(z,\ t)\,dz = 0. \tag{10}$$

The relations (9) will be expressed as

$$\bar\sigma(\bar z,\ \bar t) = \begin{cases} b(1 + t_1 - t_1 b)\,\bar\varepsilon + bWt_1\dot{\bar\varepsilon} + 1 + \dfrac{b}{W}\exp\left(\dfrac{-tb}{W}\right)\times \\ \times[1 - (1 + t_1)b + t_1 b^2]\left[\int_0^{\bar t} \bar\varepsilon \exp\left(\dfrac{\tau b}{W}\right)d\tau - \dfrac{W}{b}\right], \quad \bar z < \bar z_*, \\ e[\bar\varepsilon + 1 - \bar\Theta(\bar z)], \quad \bar z_* < \bar z < 1, \end{cases} \tag{11}$$

with the following designations:

$$\bar\sigma = \frac{3B\sigma}{A\Theta_0}, \quad \bar\varepsilon = \frac{3\varepsilon - \Theta_0}{\Theta_0}, \quad \bar t = \frac{t}{t_\text{к}}, \quad \bar z = \frac{z}{h}, \quad W = \frac{\tau}{t_\text{к}},$$

$$t_1 = \frac{\tau_0}{\tau}, \quad e = \frac{E(4G + 3K)}{18KG(1 - \nu)}, \quad b = \frac{(4G + 3K)\tau}{4G\tau + 3K\tau_0},$$

$$\bar\Theta(\bar z) = \Theta/\Theta_0 = (1 + V(z)/V_0)^{-1}.$$

In the expression (11) the variables will be changed, time will be substituted by $\xi_* = 1 - \bar z_*$. Integration of Eq. (7) gives the relation $\zeta_* = (\bar t) = \sqrt{t}$, $\dot\zeta_* = (2\zeta_*)^{-1}$. Then, Eq. (11) will have the form

$$\bar\sigma(\zeta,\ \zeta_*) = \begin{cases} b(1 + t_1 - t_1 b)\,\bar\varepsilon + bWt_1\dot\zeta_*\bar\varepsilon' + 1 + \\ + \dfrac{b}{W}\exp\left(\dfrac{-\zeta_*^2 b}{W}\right)[1 - (1 + t_1)b + t_1 b^2]\times \\ \times\left[\int_0^{\zeta_*} \dfrac{\bar\varepsilon}{\dot\zeta}\exp\left(\dfrac{\zeta^2 b}{W}\right)d\zeta - \dfrac{W}{b}\right], \quad \zeta_* < \zeta < 1, \\ e[\bar\varepsilon + 1 - \bar\Theta(\zeta)], \quad \zeta < \zeta_*, \end{cases} \tag{12}$$

where

$$\bar\Theta(\zeta) = \frac{\zeta}{\zeta + s}, \quad s = \frac{h}{2t_\text{к}V_0}.$$

The strains $\bar\varepsilon(\zeta_*)$ will be found from the boundary condition (10). With the expression (12) taken into consideration, we have

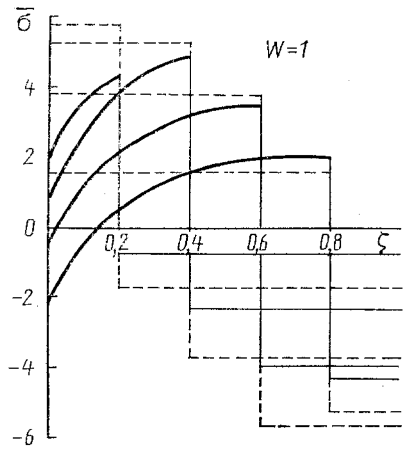
$$(1 - \zeta_*)\Big\{b(1 + t_1 - t_1 b)\,\bar\varepsilon + bWt_1\dot\zeta_*\bar\varepsilon' + 1 + \frac{b}{W}\exp\left(\frac{-\zeta_*^2 b}{W}\right)\times$$

$$\times[1 - (1 + t_1)b + t_1 b^2]\left[\int_0^{\zeta_*} \frac{\bar\varepsilon}{\dot\zeta}\exp\left(\frac{\zeta^2 b}{W}\right)d\zeta - \frac{W}{b}\right]\Big\} + \tag{13}$$

$$+ e\left[\zeta_*(\varepsilon + 1) - \int_0^{\zeta_*} \bar\Theta(\zeta)\,d\zeta\right] = 0,$$

where

$$\int_0^{\zeta_*} \bar\Theta(\zeta)\,d\zeta = \zeta_* - s\ln\left(\frac{\zeta_* + s}{s}\right).$$



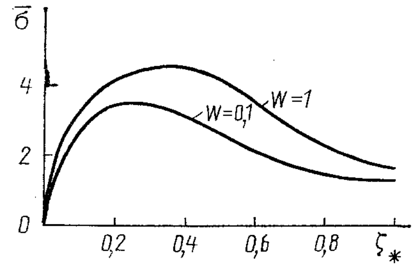


Fig. 1 Fig. 2

Fig. 1. The stress distribution across the plate in dimensionless variables with different locations of the evaporation front; solid line refers to s = 0.1, dashed, to s = 0.

Fig. 2. Variation of the tensile stress at the evaporation front during drying in terms of dimensionless variables.

The integrodifferential equation (13) can be reduced to a second-order linear differential equation by dividing by $(1 - \zeta_*)\exp(-\zeta_*^2 b/W)$ and differentiating with respect to ζ_*. These steps will result in

$$t_1\bar{\varepsilon}'' + \left[2\zeta_*\left(\frac{1+t_1}{W} + \frac{e\zeta_*}{bW(1-\zeta_*)}\right) - \frac{t_1}{\zeta_*}\right]\bar{\varepsilon}' + \frac{4\zeta_*^2}{W^2}\left[1 + \frac{e\zeta_*}{1-\zeta_*} + \frac{eW(1-\zeta_*)^{-2}}{2b\zeta_*}\right]\bar{\varepsilon} = \frac{4e\zeta_*^2}{W^2}\times \left\{\frac{2b\zeta_*(1-\zeta_*)+W}{2b\zeta_*(1-\zeta_*)^2}[\zeta_* - s\ln(\zeta_*+s)] + \frac{W(1-\zeta_*)^{-1}}{2b(\zeta_*+s)}\right\}. \quad (14)$$

The initial conditions (8) with the respective designations will have the form

$$\bar{\varepsilon}(0) = -1, \quad \bar{\varepsilon}'(0) = 0. \quad (15)$$

The problem (14), (15) was solved numerically, and then stresses were calculated. The stress level and behavior were determined with various values of the main dimensionless parameters W and s (W is the drying rate, s is the shrinkage decrease rate). It should be noted that $s = 0$ occurs in the case in which shrinkage is independent of the evaporation front V.

As the evaporation front moves deeper, its velocity V decreases, increasing the material shrinkage in accordance with Eq. (6). Thus, dried material has inhomogeneous structure across the plate. Its density increases from the surface toward the center. Since the particles stay in the evaporation zone for a short time, in the surface layer they do not have enough time to come close to one another and form a structure with maximum porosity. Then, cracks can appear.

Figure 1 shows the stress distribution across the plate with various positions of the evaporation front ζ_*. One can see from the figure that stresses have a discontinuity at the evaporation front. At the evaporation front the tensile stresses at $s = 0.1$ have monotonic behavior in time (Fig. 2). Increasing in the initial process, they attain a maximum with a certain value $\zeta_* = \zeta_{*m}$, decreasing to a suitable value in the end. As the drying rate w increases, the quantity ζ_{*m} shifts inside the plate, and the tensile stresses rise (see Fig. 2).

The dependence of the shrinkage Θ on the front velocity leads to three consequences: (a) there are residual stresses, which are tensile in the center and compressing at the periphery; (b) in the drying process, apart from tensile stresses, compressing stresses appear in the dry region (see Fig. 1); (c) the structure across the plate is inhomogeneous.

NOTATION

[D], effective vapor transfer coefficient in porous medium; E, elasticity modulus with extension and compression; G, shear elasticity modulus; h, half thickness of the plate; j, vapor flow; K, volume expansion modulus; M, molecular mass of vapor; P_s, saturated vapor pressure; R, universal gas constant; T, temperature; t, t_k, current time and drying time, respectively; V, evaporation front velocity; x, y, z, Cartesian coordinates; z_*, evaporation front coordinate; δ_{ij}, Kronecker's delta; ε_{ij}, strain tensor components; Θ, shrinkage; ν, Poisson coefficient; ρ, liquid density; σ_{ij}, stress tensor components; τ, τ_0, shear and volume relaxation time, respectively.

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